



Maths Studies Mini Topic Exams


Logic, Sets & Probability

Solutions

Video tutorials to these exam questions can be found at:

<https://www.revisionvillage.com/mini-topic-exams-studies-logic-sets-probability>

Question 1

[Maximum mark: 6] 

Consider the following propositions:

p : The cat is playing.
 q : The cat is sleeping.

- (a) Complete the following truth table. [1]

p	q	$\neg q$	$\neg q \Rightarrow p$
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	F

- (b) Write down, in words, the compound proposition $\neg q \Rightarrow p$. [2]
 (c) Write down, in symbolic form, the inverse of $\neg q \Rightarrow p$. [2]
 (d) Use your truth table to determine whether $\neg q \Rightarrow p$ is a tautology, contradiction, or neither. [1]

b) If the cat is not sleeping, then the cat is playing

c) Inverse: $q \Rightarrow \neg p$

d) Neither a Tautology or Contradiction

Logic Terms

\neg Negation 'not'

\Rightarrow Implication
'if __, then __'

Inverse of $p \Rightarrow q$ is $\neg p \Rightarrow \neg q$

Tautology: All TT values are True

Contradiction: All TT values are False

Truth Tables


\Rightarrow

P	q	$P \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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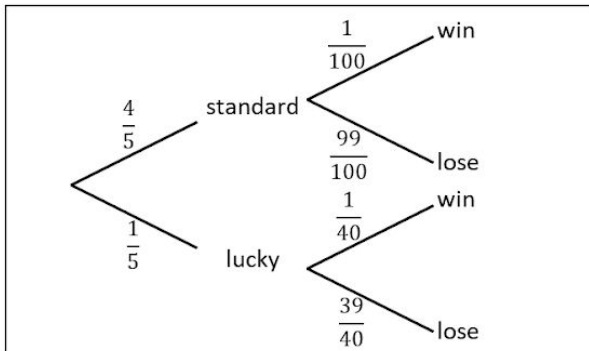
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Question 2

[Maximum mark: 6] 

In a college lottery, there are two types of tickets: 'standard' tickets and 'lucky' tickets. Every student is given one ticket at random. The total number of the standard tickets is four times greater than the number of lucky tickets.

The probability of winning the lottery with a standard ticket is $\frac{1}{100}$ and for the lucky ticket is $\frac{1}{40}$. The tree diagram below illustrates this.



- (a) Calculate the probability that the student will win the lottery. [2]
 (b) Calculate the probability that the student will get the lucky ticket and will lose the lottery. [2]
 (c) Calculate the probability that, given the student wins the lottery, he gets the standard ticket. [2]

Conditional Probability
 $P(A|B) = \frac{P(A \cap B)}{P(B)}$
 given \rightarrow

$$\begin{aligned} \text{a) } P(W) &= \left(\frac{4}{5} \times \frac{1}{100}\right) + \left(\frac{1}{5} \times \frac{1}{40}\right) \\ &= \frac{13}{1000} \\ &= \mathbf{0.013} \end{aligned}$$


$$\begin{aligned} \text{b) } P(\text{Lucky} \cap L) &= \frac{1}{5} \times \frac{39}{40} \\ &= \frac{39}{200} \\ &= \mathbf{0.195} \end{aligned}$$

$$\begin{aligned} \text{c) } P(\text{Standard}|W) &= \frac{P(\text{standard} \cap W)}{P(W)} \\ &= \frac{\left(\frac{4}{5}\right) \times \left(\frac{1}{100}\right)}{\left(\frac{13}{1000}\right)} \\ &= \frac{\mathbf{8}}{\mathbf{13}} \\ &\approx \mathbf{0.615} \end{aligned}$$

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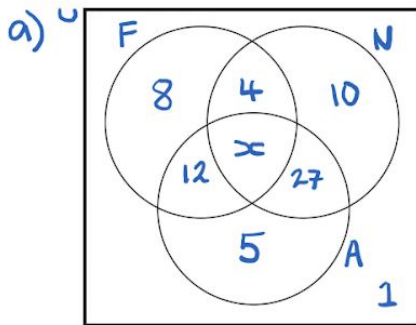
Question 3

[Maximum mark: 14] 

A group of 80 students are asked which books they read from a choice of fiction books, non-fiction books and autobiographies. The results are as follows.

- N** **A** **F**
- 5 read only autobiographies
 - 8 read only fiction books
 - 10 read only non-fiction books
 - 12 read autobiographies and fiction books, **but not** non-fiction books
 - 27 read autobiographies and non-fiction books, **but not** fiction books
 - 4 read non-fiction books and fiction books, **but not** autobiographies
 - x read all three kinds of books

- (a) Represent this information on a Venn diagram. [4]
- (b) Find the value of x , if one student does not read any these book types. [2]
- (c) A student is randomly chosen from this group of 80 students. Calculate the probability that this student reads
- * (i) non-fiction books;
 - * (ii) fiction books;
 - (iii) autobiographies, given that this student reads non-fiction books. [6]
- (d) Determine whether the events in part (c)(i) and part (c)(ii) are independent. Give a reason. [2]



$$\begin{aligned} b) \quad 80 &= 8 + 4 + 10 + 12 + x + 27 + 5 + 1 \\ 80 &= 67 + x \\ x &= 13 \end{aligned}$$

$$\begin{aligned} c) \quad i) \quad P(N) &= \frac{4 + 10 + 13 + 27}{80} \\ &= \frac{54}{80} \\ &= \frac{27}{40} \end{aligned}$$

$$\begin{aligned} ii) \quad P(F) &= \frac{8 + 4 + 12 + 13}{80} \\ &= \frac{37}{80} \end{aligned}$$

$$\begin{aligned} iii) \quad P(A|N) &= \frac{13 + 27}{4 + 10 + 13 + 27} \\ &= \frac{40}{54} \\ &= \frac{20}{27} \end{aligned}$$

$$d) \quad P(A \cap B) = P(A) \times P(B)$$

$$\text{Test: } P(N \cap F) = P(N) \times P(F)$$

$$P(N \cap F) = \frac{4 + 13}{80} = \frac{17}{80}$$

$$P(N) \times P(F) = \frac{27}{40} \times \frac{37}{80} = \frac{999}{3200}$$

\therefore Since $P(N \cap F) \neq P(N) \times P(F)$, not independent

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