IB Maths HL mini Topic Exam: Algebra

Recommended Time: 40mins.  Total Mark: 46

Student Name: ____________________________ Teacher: ____________________________

Question 1

[Maximum mark: 5] 

The 1st, 5th and 13th terms of an arithmetic sequence, with common difference $d$, $d \neq 0$, are the first three terms of a geometric sequence, with common ratio $r$, $r \neq 1$. Given that the 1st term of both sequences is 12, find the value of $d$ and the value of $r$.

Working

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Question 2

[Maximum mark: 8]  

Consider the following system of equations:

\[
\begin{align*}
x + y + z &= -1 \\
4x + 2y + z &= 3 \\
9x + 3y &= \mu
\end{align*}
\]

where \( \mu \in \mathbb{R} \).

(a) Show that this system does not have a unique solution for any value of \( \mu \).  

(b)  
(i) Determine the value of \( \mu \) for which the system is consistent.  
(ii) For this value of \( \mu \), find the general solution of the system.
Question 3

[Maximum mark: 7]

Given that \((1 + x)^3(1 + px)^4 = 1 + qx + 93x^2 + \cdots + px^7\), find the possible values of \(p\) and \(q\).
Question 4

[Maximum mark: 9]

Let \( y = (x + 1)e^{-2x}, \) \( x \in \mathbb{R}. \)

(a) Find \( \frac{dy}{dx}. \) \( \quad \) [2]

(b) Using induction, prove that \( \frac{d^n y}{dx^n} = \left[ n(-2)^{n-1} + (-2)^n(x + 1) \right] e^{-2x} \) for all \( n \in \mathbb{Z}^+. \) \( \quad \) [7]

Working

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Question 5

[Maximum mark: 17]

(a) Solve the equation \( z^3 = 27 \), \( z \in \mathbb{C} \), giving your answer in the form \( z = r(\cos \theta + i \sin \theta) \) and in the form \( z = a + bi \) where \( a, b \in \mathbb{R} \). [6]

(b) Consider the complex numbers \( z_1 = -1 + i \) and \( z_2 = \frac{1}{\sqrt{2}} \left( \cos \left( \frac{\pi}{3} \right) + i \sin \left( \frac{\pi}{3} \right) \right) \).

(i) Write \( z_1 \) in the form \( r(\cos \theta + i \sin \theta) \).

(ii) Calculate \( z_1z_2 \) and write in the form \( a + bi \) where \( a, b \in \mathbb{R} \).

(iii) Hence find the value of \( \tan \left( \frac{\pi}{12} \right) \) in the form \( c + d\sqrt{3} \) where \( c, d \in \mathbb{Z} \).

(iv) Find the smallest \( p \in \mathbb{Q}^+ \) such that \( (z_1z_2)^p \) is a positive real number. [11]

Working