IB Maths HL mini Topic Exam: **Functions & Equations**

Recommended Time: 40mins.  
Total Mark: 42

Student Name: ___________________________  
Teacher: ___________________________

**Question 1**

[Maximum mark: 6]

The function $f$ is defined by $f(x) = \sqrt{2x + 1}$, $-14 \leq x \leq 13$.

(a) Write down the range of $f$.  
(b) Find an expression for $f^{-1}$.  
(c) Write down the domain and range of $f^{-1}$.

**Working**

/6
Question 2

[Maximum mark: 5]

The quadratic equation $x^2 - kx + (k - 1) = 0$ has roots $\alpha$ and $\beta$. Without solving the equation, find the possible values of the real number $k$ given that $\alpha^2 + \beta^2 = 17$. 

/5
Question 3

[Maximum mark: 6]

Given that \((x - 4)\) is a factor of \(f(x) = x^3 - 2x^2 + ax + b\) and that division of \(f(x)\) by \((x + 2)\) leaves a remainder of 18, find the value of \(a\) and the value of \(b\).
Question 4

[Maximum mark: 8]

Let \( p(x) = \frac{1}{4}x^5 - 2x^4 - 5x^3 + 40x^2 + 16x - 128, x \in \mathbb{R} \).

(a) For the polynomial equation \( p(x) = 0 \), state
   (i) the sum of the roots;  
   (ii) the product of the roots.  

A new polynomial is defined by \( q(x) = p(2x - 2) \). For the polynomial equation \( q(x) = 0 \), find

(b) (i) the sum of the roots;  
   (ii) the product of the roots.

Working
Question 5

[Maximum mark: 17]

The function \( f \) is defined by \( f(x) = 1 + \frac{4x}{x + 3}, \ x \in \mathbb{R}, \ x \neq -3. \)

(a) Sketch the graph of \( y = f(x) \), indicating clearly any asymptotes and points of intersection with the \( x \) and \( y \) axes. \([4]\)

(b) Find an expression for \( f^{-1} \). \([4]\)

(c) Find all values of \( x \) for which \( f(x) = f^{-1}(x) \). \([3]\)

(d) Solve the inequality \( |f(x)| < 2 \). \([4]\)

(e) Solve the inequality \( f(|x|) < 2 \). \([2]\)

Working

More working space over page