IB Maths SL mini Topic Exam: Algebra

Recommended Time: 40mins.  Total Mark: 49

Student Name: ______________________  Teacher: __________________

Question 1

[Maximum mark: 7]  X

An arithmetic sequence is given by 3, 5, 7, ...

(a) Write down the value of $d$.  [1]

(b) Find.

(i) $u_{10}$.  

(ii) $s_{10}$.  [4]

(c) Given that $u_n = 253$, find the value of $n$.  [2]

Working

/7
Question 2

[Maximum mark: 6]  

(a) Write down the value of.
   
   (i) \( \log_2 8 \)  
   
   (ii) \( \log_5 \left( \frac{1}{25} \right) \)  
   
   (iii) \( \log_9 3 \)  

   [3]

(b) Hence, solve \( \log_2 8 + \log_5 \left( \frac{1}{25} \right) + \log_9 3 = \log_{16} x \).  

   [3]
Question 3

[Maximum mark: 7]

In the expansion of \((2x + 1)^n\), the coefficient of the term in \(x^2\) is \(40n\), where \(n \in \mathbb{Z}^+\). Find \(n\).

Working

\[
\binom{n}{2} (2x)^2 = 40n
\]

\[
\frac{n(n-1)}{2} \cdot 4x^2 = 40n
\]

\[
2n(n-1)x^2 = 40n
\]

\[
x^2 = 20
\]

\[
n(n-1) = 20
\]

Factors of 20: 1, 2, 4, 5, 10, 20

The only pair that satisfies this is: \(n = 5\) and \(n-1 = 4\) or vice versa.

\[
\text{Therefore, } n = 5
\]
Question 4

[Maximum mark: 14]

The first two terms of an infinite geometric sequence, in order, are $3\log_3 x$, $2\log_3 x$, where $x > 0$.

(a) Find $r$. [2]

(b) Show that the sum of the infinite sequence is $9\log_3 x$. [3]

The first three terms of an arithmetic sequence, in order, are

$$\log_3 (x), \log_3 \left(\frac{x}{3}\right), \log_3 \left(\frac{x}{9}\right),$$

where $x > 0$.

(c) Find $d$, giving your answers as an integer. [3]

Let $S_6$ be the sum of the first 6 terms of an arithmetic sequence.

(d) Show that $S_6 = 6\log_3 (x) - 15$. [3]

(e) Given that $S_6$ is equal to one third of the sum of the infinite geometric sequence, find $x$, giving your answer in the form of $3^p$. [3]

Working

More working space over page
Question 5

[Maximum mark: 15]

The first three terms of an infinite geometric sequence are $k-4$, $4$, $k+2$, where $k \in \mathbb{Z}$.

(a) (i) Write down an expression for $r$.

(ii) Hence, show that $k$ satisfies the equation $k^2 - 2k - 24 = 0$ [5]

(b) (i) Find the possible values for $k$.

(ii) Find the possible values for $r$. [5]

(c) The geometric sequence has a finite sum.

(i) Which value of $r$ leads to this sum. Justify answer.

(ii) Find the sum of the sequence. [5]