Mathematics: Applications & Interpretation SL & HL

1 Page Formula Sheet – First Examinations 2021 – Updated Version 1.1

Prior Learning SL & HL

Area: Parallelogram	$A = bh$, b = base, h = height
Area: Triangle	$A = \frac{1}{2}(bh)$, b = base, h = height
Area: Trapezoid	$A = \frac{1}{2}(a+b)h$, a, b = parallel sides, h = height
Area: Circle	$A = \pi r^2$, r = radius
Circumference: Circle	$C = 2\pi r$, r = radius
Volume: Cuboid	$V = lwh$, l = length, w = width, h = height
Volume: Cylinder	$V = \pi r^2 h$, r = radius, h = height
Volume: Prism	$V = Ah$, A = cross-section area, h = height
Area: Cylinder curve	$A = 2\pi rh$, r = radius, h = height
Distance between two points (x_1, y_1) , (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of midpoint	$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, for endpoints $(x_1, y_1), (x_2, y_2)$

Prior Learning HL only

Solutions of a quadratic equation in the form $ax^2 + bx + c = 0$	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$

Topic 1: Number and algebra - SL & HL

The nth term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
Sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
The nth term of a geometric sequence	$u_n = u_1 r^{n-1}$
Sum of n terms of a finite geometric seq.	$S_n = \dfrac{u_1(r^n - 1)}{r - 1} = \dfrac{u_1(1 - r^n)}{1 - r}$, $r \neq 1$
Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ FV is future value, PV is present value, n is the number of years, k is the number of compounding periods per year, r% is the nominal annual rate of interest
Exponents & logarithms	$a^x = b \Leftrightarrow x = \log_a b$, $a, b > 0, a \neq 1$
Percentage error	$\varepsilon = \left\lvert \dfrac{v_A - v_E}{v_E} \right\rvert \times 100\%$ v_A = approximate value, v_E = exact value

Topic 1: Number and algebra - HL only

Laws of logarithms for $a, x, y > 0$	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$
The sum of an infinite geometric sequence	$S_\infty = \dfrac{u_1}{1 - r}$, $\lvert r \rvert < 1$
Complex numbers	$z = a + bi$
Discriminant	$\Delta = b^2 - 4ac$
Modulus-argument (polar) & Exponential (Euler) form	$z = r(\cos\theta + i\sin\theta) = re^{i\theta} = r\mathrm{cis}\theta$
Determinant of a 2×2 matrix	$\boldsymbol{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \det \boldsymbol{A} = \lvert \boldsymbol{A} \rvert = ad - bc$
Inverse of a 2×2 matrix	$\boldsymbol{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \boldsymbol{A}^{-1} = \frac{1}{\det \boldsymbol{A}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$
Power formula for a matrix	$\boldsymbol{M}^n = \boldsymbol{P}\boldsymbol{D}^n\boldsymbol{P}^{-1}$, where $\boldsymbol{P}$ is the matrix of eigenvectors and $\boldsymbol{D}$ is the diagonal matrix of eigenvalues

Topic 2: Functions – SL & HL

Equations of a straight line	$y = mx + c$; $ax + by + d = 0$; $y - y_1 = m(x - x_1)$
Gradient formula	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
Axis of symmetry of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow x = -\dfrac{b}{2a}$

Topic 2: Functions – HL only

Logistic function	$f(x) = \dfrac{L}{1 + Ce^{-kx}}$, $L, k, C > 0$

Topic 3: Geometry and trigonometry – SL & HL

Distance between 2 points (x_1, y_1, z_1) , (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
Coordinates of midpoint of a line with endpoints (x_1, y_1, z_1) , (x_2, y_2, z_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
Volume: Right-pyramid	$V = \frac{1}{3}Ah$, A = base area, h = height
Volume: Right cone	$V = \frac{1}{3}\pi r^2 h$, r = radius, h = height
Area: Cone curve	$A = \pi rl$, r = radius, l = slant height
Volume: Sphere	$V = \frac{4}{3}\pi r^3$, r = radius
Surface area: Sphere	$A = 4\pi r^2$, r = radius
Sine rule	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$ $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$
Area: Triangle	$A = \frac{1}{2}ab\sin C$
Length of an arc	$l = \dfrac{\theta}{360} \times 2\pi r$ θ = angle in degrees, r = radius
Area of a sector	$A = \dfrac{\theta}{360} \times \pi r^2$ θ = angle in degrees, r = radius

Topic 3: Geometry and trigonometry – HL only

Length of an arc	$l = r\theta$ r = radius, θ = angle in radians
Area of a sector	$A = \frac{1}{2}r^2\theta$ r = radius, θ = angle in radians
Identities	$\cos^2\theta + \sin^2\theta = 1$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
Transformation matrices	$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$, reflection in the line $y = (\tan\theta)x$ $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$, horizontal stretch by scale factor of k $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$, vertical stretch with scale factor of k $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$, centre $(0,0)$, enlargement with scale factor of k $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, anticlockwise rotation of angle θ about the origin $(\theta > 0)$ $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, clockwise rotation of angle θ about the origin $(\theta > 0)$
Magnitude of a vector	$\lvert \boldsymbol{v} \rvert = \sqrt{{v_1}^2 + {v_2}^2 + {v_3}^2}$
Vector equ. of a line	$\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$
Parametric form of the equation of a line	$x = x_0 + \lambda l,\ y = y_0 + \lambda m,\ z = z_0 + \lambda n$
Scalar product	$\boldsymbol{v} \cdot \boldsymbol{w} = v_1w_1 + v_2w_2 + v_3w_3$ $\boldsymbol{v} \cdot \boldsymbol{w} = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert \cos\theta$ where θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$
Angle between two vectors	$\cos\theta = \dfrac{v_1w_1 + v_2w_2 + v_3w_3}{\lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert}$
Vector product	$\boldsymbol{v} \times \boldsymbol{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$ $\lvert \boldsymbol{v} \times \boldsymbol{w} \rvert = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert \sin\theta$ where θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$
Area of a parallelogram	$A = \lvert \boldsymbol{v} \times \boldsymbol{w} \rvert$, where $\boldsymbol{v}$ and $\boldsymbol{w}$ form two adjacent sides of a parallelogram

Topic 4: Statistics and probability - SL & HL

Interquartile range	$\mathrm{IQR} = Q_3 - Q_1$
Mean, $\bar{x}$, of a set of data	$\bar{x} = \dfrac{\sum_{i=1}^{k} f_i x_i}{n}$, where $n = \sum_{i=1}^{k} f_i$
Probability of an event A	$\mathrm{P}(A) = \dfrac{n(A)}{n(U)}$
Complementary events	$\mathrm{P}(A) + \mathrm{P}(A') = 1$
Combined events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
Mutually exclusive events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$
Conditional probability	$\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
Independent events	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$
Expected value of a discrete random variable X	$\mathrm{E}(X) = \sum x\,\mathrm{P}(X = x)$
Binomial distribution Mean ; Variance	$X \sim \mathrm{B}(n, p)$ $\mathrm{E}(X) = np$; $\mathrm{Var}(X) = np(1 - p)$

Topic 4: Statistics and probability – HL only

Linear transformation of a single random variable	$E(aX + b) = a\mathrm{E}(X) + b$ $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$
Linear combinations of n independent random variables, $X_1, X_2, \ldots, X_n$	$\mathrm{E}(a_1X_1 \pm a_2X_2 \pm \ldots \pm a_nX_n) = a_1\mathrm{E}(X_1) \pm a_2\mathrm{E}(X_2) \pm \ldots \pm a_n\mathrm{E}(X_n)$ $\mathrm{Var}(a_1X_1 \pm a_2X_2 \pm \ldots \pm a_nX_n) = {a_1}^2\mathrm{Var}(X_1) + {a_2}^2\mathrm{Var}(X_2) + \cdots + {a_n}^2\mathrm{Var}(X_n)$
Unbiased estimate of population variance	$s_{n-1}^2 = \dfrac{n}{n-1} s_n^2$ Sample statistics
Poisson distribution Mean ; Variance	$X \sim \mathrm{Po}(m)$ $\mathrm{E}(X) = m$; $\mathrm{Var}(X) = m$
Transition matrices	$\boldsymbol{T}^n \boldsymbol{s}_0 = \boldsymbol{s}_n$, where $\boldsymbol{s}_0$ is the initial state

Topic 5: Calculus - SL & HL

Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
Integral of x^n	$\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$, $n \neq -1$
Area enclosed by a curve and the x-axis	$A = \int_a^b y\,dx$, where $f(x) > 0$
The trapezoidal rule where $h = \frac{b-a}{n}$	$\int_a^b y\,dx \approx \frac{1}{2}h((y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1}))$

Topic 5: Calculus – HL only

Derivative of $\sin x$	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
Derivative of $\tan x$	$f(x) = \tan x \Rightarrow f'(x) = \dfrac{1}{\cos^2 x}$
Derivative of e^x	$f(x) = e^x \Rightarrow f'(x) = e^x$
Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \dfrac{1}{x}$
Chain rule	$y = g(u)$, $u = f(x) \Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
Product rule	$y = uv \Rightarrow \dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
Quotient rule	$y = \dfrac{u}{v} \Rightarrow \dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
Standard integrals	$\int \frac{1}{x} dx = \ln\lvert x \rvert + C$ $\int \sin x\,dx = -\cos x + C$ $\int \cos x\,dx = \sin x + C$ $\int \frac{1}{\cos^2 x} dx = \tan x + C$ $\int e^x dx = e^x + C$
Area enclosed by a curve and x or y-axes	$A = \int_a^b \lvert y \rvert\,dx$ or $A = \int_a^b \lvert x \rvert\,dy$
Volume of revolution about x or y-axes	$V = \int_a^b \pi y^2\,dx$ or $V = \int_a^b \pi x^2\,dy$
Acceleration	$a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}^2s}{\mathrm{d}t^2} = v\dfrac{\mathrm{d}v}{\mathrm{d}s}$
Distance; Displacement travelled from t_1 to t_2	$\mathrm{dist} = \int_{t_1}^{t_2} \lvert v(t) \rvert\,dt$; $\mathrm{disp} = \int_{t_1}^{t_2} v(t)\,dt$
Euler's method	$y_{n+1} = y_n + h \times f(x_n, y_n)$; $x_{n+1} = x_n + h$ where h is a constant (step length)
Euler's method for coupled systems	$x_{n+1} = x_n + h \times f_1(x_n, y_n, t_n)$ $y_{n+1} = y_n + h \times f_2(x_n, y_n, t_n)$ $t_{n+1} = t_n + h$ where h is a constant (step length)
Exact solution for coupled linear differential equations	$\boldsymbol{x} = Ae^{\lambda_1 t}\boldsymbol{p}_1 + Be^{\lambda_2 t}\boldsymbol{p}_2$